

Differential Equations

SIMIODE Learning Environment Index

Index of Modeling Scenarios and Text Narratives
Numbered materials have been refereed, revised, and posted.

This Index is meant to simulate the topics found in a traditional differential equations course.
The numbering system reflects chapter sequencing.

These materials are hyper-linked to Student Versions under Resources
at <https://www.simiode.org/>
and they are available to the public with no sign in or membership.

User must join and be [registered](#) and logged in to Teachers Group
to see Extended Teacher Versions.

Those materials not hyper-linked are under development.

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and on Modeling Section heading to return to Table of Contents.**

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APPENDIX

Strategies for Analytic Solutions and Qualitative Analysis

[First Order Differential Equations](#)

[Second Order Differential Equations](#)

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Modeling One - First Order Differential Equations

Narrative and Development

Modeling Death with M&M 's and Simulation

Immigration Model with M&M's

Modeling Change Discretely

Moving to Differential Equation Model

Introducing CAS solving with modest analytic solutions in appendices as needed

1-001-Text-SepVariables

Technique narrative on solution method of separation of variables

1-002-Text-Integrating Factor

Technique narrative on solution method of integrating factors

1-009-Text-Bifurcation

Early introduction to bifurcation with experimentation.

Activities

1-001-M&MDeathImmigration

Modeling Death and Immigration with M&M 's and Simulation

1-001A-M&MDeathImmigration-Variation

Variation on modeling death and immigration with M&M's

1-001B-M&MDeathImmigrationMystery

Each student sets own immigration rate and others need to solve this mystery

1-001pgf- BirthDeathImmigration

Probability generating function approach to simulation.

1-002-Tossing

Modeling a simulation of a large number of dice tossings

1-003-CollegeSavings

Saving for Child's College Education

1-004-Microorganism Immigration

Modeling Immigration in a Petri Dish

Evaporation and Radioactive Decay Modeling

Modeling Falling Stack of Coffee Filters

Modeling Sphere of Salt in Solution

1-005-OilSlick

Modeling the Spread of Oil Slick with Incomplete Data

1-005A-OilSlick

Same as 1-5-OilSlick but with client setting.

1-005a-ChemDataCollection

Analysis of incomplete reaction data

1-005b-ChemDataCollection

Analysis of incomplete reaction data

1-006-FinancingSavingsAndLoan

Bank Investment Analysis and Bank Loan Analysis

1-007-AntTunnelBuilding

How long does it take an ant to build a tunnel of length x in soil?

1-009-ICUSpread

Modeling the spread of ICU's in US Hospitals from 1958-1974

1-010-LSDAndProblemSolving

Modeling LSD in the body and correlating amounts with test performance.

1-011-Kinetics

Chemical Kinetics Models - Zeroth, First, and Second Order Reactions

1-011A-Kinetics

Same as 1-11-Kinetics but with more guidance and less narrative.

1-012-SublimationCarbonDioxide

Sublimation of Carbon Dioxide

1-013-Sleuthing

Situations associated with stopping cars, projectile steel ball, and time of death

1-014-DrainingContainers

For fixed volume column which radius of cylinder of water drains fastest

1-015-Torricelli

Modeling the Emptying of a Column of Water

1-016-DogDrugs

Modeling drugs for anesthesiology

1-017-DiseaseSpread

Modeling spread of disease using logistic equation

1-018-LogisticPopModeling

Limited Growth Population Modeling

1-019-RocksInTheirHeads

Sensitivity Analysis - Data Collection Experiment Comparing of Rock Masses

1-020-IceMelt

Analyzing models on which melts first a sphere or cube of ice of the same volume

1-021-FeralCatControl

Model for three feral cat control policies

1-022-SpreadOfTechnology

Model the spread of a number of technological advances

1-023-RumorSpread

Rate of Spread of False and True articles on the Internet

1-024-MalariaControl

Modeling and numerical methods for first order malaria growth

1-025-MixingItUp

Modeling more and more complex salt mixing situations.

1-026-Evaporation

Modeling the evaporation of an alcohol and water mixture in various containers.

1-027-StochasticProcesses

Modeling randomness with stochastic processes.

1-028-SouthernSweetIcedTea

Data is offered to model making sweet iced team using luminescence

1-029-ConeToCubeFlow

Modeling water flowing from cone to cube and out

1-030-IntraocularGasBubbles

Modeling dissipation of intraocular gas bubbles used in eye surgery.

1-031-CoolIt

Changing temperature of container of water in a changing environment.

1-032-WordPropagation

Modeling the rate at which words propagate through English language text

1-033-SouthernBarbeque

Phases of barbequing brisket are modeled using real data.

1-034-FishMixing

Student designed fishing strategies for mix of fish in lake

1-036-NeutralBuoyancy

Finding depth in water at which an object settles to neutral buoyancy

1-037-CommonColdSpread

Students conduct simulation of spread of common cold and model.

1-038-Ebola

Modeling Ebola epidemic with first order differential equation models

1-040-OutcomeSavings

Determining monthly deposit rate for long term savings goal

1-041-AirToTop

Variable ascent rate and air management in SCUBA diving

1-042-Kool-Aid

Modeling the amount of drink powder in a second tank of flow system.

1-043-CoolingUpAndDown

Air conditioning cooling modeling.

1-044-CollegeBound

Planning for full college education costs for the daughter of a friend.

1-045-TimeOfDeath

Determining time of death given observations and environmental conditions.

1-050-BargeAhead

Optimizing a barge trip upriver

[1-051-OneTankSaltModel](#)

Build one compartment salt mixing model

[1-052-SaltWaterTanks](#)

Studying tank in which water inflow containing salt increases.

[1-055-WaterFallingInCone](#)

Modeling the falling of water in a right circular cone

[1-057-FiguringFluidFlow](#)

Evaluating three models of fluid flow from a tank using data

[1-058-WaterClocks](#)

A container is designed so water will fall out at constant rate of change in height.

[1-059-ContainerShapeFallingWater](#)

Modeling column of falling water in different shaped containers.

[1-060-SalesMarketing](#)

Building a model of sales of consumer products from a classic marketing study.

[1-064-TorricelliBox](#)

Modeling falling column of water with a box at the base of the column

[1-061-PotatoCooling](#)

We model cooling of a baked potato and compare it to student-collected data.

[1-066-USCensusModeling](#)

Modeling the US Census data with several different models.

[1-070-FisheryHarvest](#)

Modeling harvesting of Atlantic cod fishery

[1-071-S-NewtonWatsonTimeOfDeath](#)

Sherlock Holmes determines time of death

[1-073-WaterExitBottle](#)

Estimating a parameter in Torricelli's model of water exiting a container

[1-074-BottleWaterFlow](#)

Comparing two models of water flowing out of a container through exit hole

[1-079-HomeHeating](#)

Modeling how to heat your home while you are away

[1-080-DrugAdministration](#)

Building a simple model for drug administration

[1-081-TumorGrowth](#)

Two different models for growth of cancer tumor.

[1-085-DrugBolus](#)

Modeling intravenous bolus of drug in the body

[1-086-MedicinalPill](#)

Modeling administration of medicinal pills.

[1-090-EmptySphericalTank](#)

Comparing two ways to empty spherical tank of water

[1-091-Slopefields](#)

Building population models for various situations and using slope fields

[1-093-SucroseReaction](#)

Determining model for sucrose hydrolysis using lab data

[1-094-SteepingTea](#)

Modeling temperature change and dissolution of sugar in brewing fruit tea

[1-095-RatingChessPlayers](#)

Using Elo's Method for rating chess players and difference equations

[1-101-ClassM&MDeathImmigration](#)

Generating data and using individual model to estimate parameters

[1-105-AnimalFalling](#)

Comparing terminal velocity for variety of animal's falling

[1-115-ModelingWithFirstOrderODEs](#)

Several short illustrations and two exercises for modeling

[1-120-CircularRollerCoaster](#)

Modeling a circular roller coaster to determine velocity to stay on track

[1-122-SpreadPEV](#)

Using recent sales data model the spread of plug-in electric vehicles.

[1-140-LeakyBucket](#)

Modeling the height of water in a tank with a leak and water pouring in

Modeling OneA - Numerically Solving Differential Equations

Narrative and Development

Differential Equations Which Are Just Impossible!

Simple Step Iterating Method

Euler's Method

Numerical Experiments

Improving on Euler's Method

Comparison of Methods with CAS DE solver approach

Activities

These are embedded in the development of the numerical methods.

Modeling Two - Limited Growth Models

Narrative and Development

Exponential Growth Population Model

Limited Growth Population Modeling

Logistic Model Development

Estimating Parameter Strategies for Logistic Growth Model

Activities

Comparison with Transformed Data vs. Direct Optimization of Sum of Square Error

Approaches for Logistic Growth in Parameter Estimation

Simulation with M&M's of Logistic Growth Model for Spread of Disease

Analysis of "Struggle for Existence" Data from G. F. Gause, Soviet Ecologist, From 1930's

Modeling the Spread of Technological Innovation in the United States with Real Data

Maximum Sustainable Yield In Harvesting Models

Harvesting a Renewable Resources - Some Analysis

Running a Catfish Farm

[Modeling Three - Second Order Homogeneous Differential Equation Models](#)

Narrative and Development

Spring Mass Dashpot Modeling

Modeling with Newton's Second Law of Motion

Free Body Diagram Modeling Tool

Using Initial Condition Information for Completion of Solution

General Forms of Solutions

Characteristic Equation and Roots and Their Significance

Complex Roots to Characteristic Equation and Their Meaning

Various Types of Damping - Under, Critical, and Over

Repeated Roots for Characteristic Equation - Self-Discovery

Bookkeeping and Rearrangements - Phase Angle Issues

Activities

[3-001-SpringMassDataAnalysis](#)

Data on a spring mass system with resistance is given for modeling for analysis

[3-002-ModelsMotivatingSecondOrder](#)

From real data several ways to model spring mass system emerge.

Estimating Resistance Parameter in Spring Mass Dashpot Modeling Design

Modeling Parachuting

Modeling Frequency of Spring Mass Dashpot Motion

[3-004-VanderPol](#)

Study of van der Pol's equation with applications and spreadsheet simulation

[3-006-Buoyancy](#)

Data on a bobbing container motivates model and parameter estimation

[3-008-HangTime](#)

Hang Time Modeling

[3-010-EnergyInSpringSystems](#)

Exploring damping and forcing terms to discover energy in system

Logarithmic Decrement

Bad News - Positive Real Part of Characteristic Root

[3-013-WhiffleBallFall](#)

Using data on whiffle ball fall model resistance and predict the fall position.

Rocket Thrust Modeling

Inverse Problems - Ascertaining Parameters Given Observational Data

[3-016-FallingCoffeeFilters](#)

Using data on stack of coffee filters to build model and estimate parameters.

[3-017-StackedCoffeeFilters](#)

Using data on stacked coffee filter falling from the literature build models.

3-019-ShuttleCock Fall

Modeling a falling shuttlecock

Keeping Costs Down - Manufacturing Cheaper Springs with High Performance

RLC Circuits - Basic Circuit Notions and Second Order Differential Equation Models

Swinging Along - Modeling, Massless and Physical Pendula

Pendulum Motion and Logarithmic Decrement

3-029-FerrisWheelCatch

We model the throw of an object to a person on a moving Ferris wheel.

3-030-SecondOrderIntro

Broad introduction to second order homogeneous and nonhomogeneous, linear, constant coefficient, differential equations with many applications

3-035-StadiumDesign

Design a stadium which is fair to home run hitters in all directions.

3-040-FirstPassageTime

We model and determine the first passage time for underdamped oscillator

3-041-UpDown

Relate times when projectile passes the same point – up and then down

3-044-DeepWell

Given total time of pebble fall to sound coming back tell how deep a well

3-045-RampBounce

Bounce a ball on a tilted ramp for optimal horizontal distance traversed

3-054-Relay

Place infielder in optimal position for minimum time relay through form outfield

3-065-UpDown

Modeling vertical projectile motion with resistance to address some issues.

3-070-FallingInWater

Drop a canister in column of water, collect data from video, model motion

3-075-RLCCircuits

An introduction to RLC circuits is offered including definitions and modeling.

3-080-PendulumModeling

Several different pendulum configurations are modeled and compared.

3-085-SimplePendulum

Modeling pendulum motion and verification of period with data

3-090-OneMassSpring

Data on a single mass spring system permits modeling of oscillator

3-090-Text-ChebyshevPolynomialSolution

Small study of Chebyshev Equation for which there are polynomial solutions

3-101-SpringMassFirstTry

Modeling a simple spring mass with no damping conjecturing solutions

[3-102-SpringMassDamped](#)

Modeling a simple spring mass with damping conjecturing solutions

[3-110-MilitarySpringMassApplication](#)

Modeling the shock absorber system for an Army vehicle.

[3-140-TwoSpringOneMassFixedEnds](#)

Modeling two spring, single mass with fixed ends

[Modeling Four - Second Order NonHomogeneous Differential Equation Models](#)

Narrative and Development

Driver for Spring Mass Dashpot Modeling

Intelligent Conjecturing for NonHomogeneous Solution

Building General Solutions with Homogeneous and NonHomogeneous Parts

Transient and Steady State Portions of Solutions

Phase Angle for Solutions

Using Initial Condition Information for Completion of Solution

General Forms of Solutions

Activities

RLC Tuner Circuit to Build a Radio

Parameter Estimation Through Steady State Data

Forced Vibration and No Damping

Frequency Response - Maximum Steady State Amplitude

[4-020-AnIEDBlast](#)

Modeling the effects of an Improvised Explosive Device

[4-023-MysteryCircuit](#)

Students assigned various input voltages to a circuit to see what the circuit is.

Building Swaying

Killer Speed Bumps

[4-035-ParEstSteadyState](#)

Input Output Analysis analyzing steady state to estimate parameters.

[4-050-ResonanceBeats](#)

We study the notions of resonance and beats for undamped system

[4-060-CircuitTuner](#)

Building the differential equation for a radio tuner.

Modeling Five - Linear Systems of Differential Equation Models

Narrative and Development

Intelligent Conjecturing for Homogeneous Solution

Eigenvalues - Voodoo Dispelled

Converting Second Order to System and Meaning of Eigenvalues and Eigenvectors.

Building General Solutions with Homogeneous and NonHomogeneous Parts

Time Out for Small Motor Skills and Eye Hand Coordination Practice

Trial Run at Two Solution Strategies

Two Compartment Model Analyses

Activities

5-005-Dialysis

Modeling Dialysis Machine

5-007-ChemOpt

Optimization for a Chemical Reaction

5-0012-LipoproteinMetabolism

Medical study data to build and affirm build a model for low-density-lipoprotein.

Tuned Mass Dampers

5-014-TwoSpringMass

Build Free Body Diagram and model for two spring configuration.

5-022- ColdPill

Modeling flow of drug from gastrointestinal tract to bloodstream for peak

Optimization for a Chemical Reaction

5-028-SaltCompartments

Amount of salt in two water tanks is modeled in several ways

5-030-AirshedSulphur

Analyzing a model of the production of sulphur compounds in a Montana airshed.

5-040-TunedMassDampers-Part I

Applying a second mass to keep structure from experiencing resonance

5-040-TunedMassDampers-Part II

Applying second mass to keep structure from large displacement

Inverse Problems - Ascertaining Parameters Given Observational Data

5-080-SpaceFlightRecolonize

Modeling the recolonization of the human race on a distant planet.

[Modeling Six - NonLinear Systems of Differential Equation Models](#)

Narrative and Development

Numerical Solution Strategy

Equilibria and Stability Analysis

Linearization and Support from Homogeneous System Analysis

Nonlinear Ecological Modeling

Predator-Prey Modeling

Mimicry in Nature

Competition Modeling

Higher Trophic Level Models

West Point Acorn, Rodent, Rattlesnake Populations

Boys Dormitory Epidemic

1914 Influenza Epidemic

Epidemic Models

Flour Beetles Predation with Hiding

Activities

[6-001-Epidemic](#)

English Boarding School NonLethal Influenza

[6-002-EulerCromerPendulum](#)

Using the study of nonlinear pendulum to implement numerical methods

[6-004-VillageEpidemic](#)

Build a model of a mid seventeenth century English village epidemic

Competition in Ecology

[6-005-InsectColonyOpt](#)

Insect Colony Optimal Control

Predator Prey Modeling with Satiation and Limited Growth

Predator Prey Modeling with Hiding

Predator Prey Modeling and Optimal Control

[6-008-PursuitModels](#)

Linearization and Support from Homogeneous System Analysis

[6-012-RiverCrossing](#)

Building a model to help cross a river with current to land at specific spot

[6-015- CombatingEbolaEpidemic](#)

Making policy recommendations from models of spread of Ebola

[6-018-ExploringSIRModel](#)

Modeling rumor and disease spread.

[6-020-AlgaePopulationSelf-Replenishment](#)

Investigate the massive algal blooms that struck Lake Chapala, Mexico

[6-021-AcornsRodentsSnakes](#)

Building a three trophic level model of acorns, rodents, and snakes

[6-023-DroneHeadingHome](#)

Moving against a headwind create model of drone flight to fixed delivery point

[6-025-WhalesAndKrill](#)

Use Excel to observe qualitative behavior a predator-prey model

[6-028-SaltCompartments](#)

Amount of salt in two water tanks is modeled when tank volumes are changing

[6-030-SaltTorricelli](#)

Modeling complex salt levels in a falling column of water.

Nonlinear Ecological Modeling

[6-035-Shampoo](#)

Modeling the amount of shampoo in a bottle during a shower

Predator- Prey Modeling

Competition Modeling

[6-040-StruggleForExistence](#)

Using historical data to model multiple species growth

[6-070-BeerBubbles](#)

Modeling the rise and size of beer bubbles in a sitting glass of beer

Modeling Seven - Changing the Venue for Solution Success - Laplace Transforms

Narrative and Development

Modeling with Spikes and Jumps

Transformations in General

General Laplace Transforms -Lots of Functions

Revisit Second Order Differential Equations and Linear Systems with Laplace Transform

Transfer Function Thinking

Solution Strategies with Laplace Transforms

Living in the Frequency Domain

Convolution Applications

Activities

7-005-LaplaceTransformOverview

Introduction and application of Laplace Transforms

7-008-MachineReplacement

Laplace Transforms - Convolution Applications - Replacement Theory

RLC Filter Circuit and Laplace Transform View Point

7-010-MultipleDoses

Modeling several multiple dose approaches for drugs is considered.

Modeling Eight - Representing Natural Phenomena with Sines and Cosines

Narrative and Development

8-002-Text-TrigSumRepresentation

Representing functions as sums of trigonometric functions.

Discovery and Least Squares Criteria

Approximating Functions with sums of Sine Functions

General Fourier Series Modeling

Activities

Creating Complex Sounds from Simple Sounds

Analyzing Signals with Simple Functions

Orthonormal Families and Their Good Times

[Modeling Nine - Modeling with Differential Equations in Higher Dimensions](#)

Narrative and Development

Partial Differential Equations

Numerical Solutions

Modeling Spread of Heat in Limited Environment - Heat Equation

Applications of the Heat Equation - Formulating Conditions

Extensions of the Heat Equations

Making Sound with Waves - Wave Equation

Modes

Analytic Solutions - Building for Success

Boundary Value Problems and Fourier Analysis - Bringing it All Together

Activities

Root Cellar Modeling with Heat Equation

Noninvasive Analysis with the Heat Equation

[9-012-PDEGuitarTuning](#)

Tuning a Stringed Instrument with the Wave Equation

Orthonormal Families and Their Good Times

[9-020-HorizontalBeam](#)

Modeling a suspended beam and collecting data to justify the model

Modeling Ten – General Modeling

APPENDIX

Strategies for Analytic Solutions and Qualitative Analysis

First Order Differential Equations

1-001-Text-SepVariables

1-002-Text-IntegratingFactor

Conjecturing

1-009-Text-Bifurcation

Second Order Differential Equations

Conjecturing and Eigenvalues

The Cases for Eigenvalues

Homogeneous and NonHomogeneous Solutions and Building Final Solutions

3-090-Text-ChebyshevPolynomialSolution

Linear Systems of Differential Equations

Equilibrium and Stability Issues

Conjecturing, Eigenvectors, and Eigenvalues

Construction of Solutions - Homogenous and NonHomogeneous Equations

5-030-Text- LinNonHomoSystemSol

Variation of Parameter Method for linear system of nonhomogeneous equations

NonLinear Systems of Differential Equations

Equilibrium and Stability Issues

Linearization and Translation

Some Special Situations - Orbits in Predator Prey Models

Fourier Series Development

8-002-Text-TrigSumRepresentation

General Formulae

Extensions - Odd and Even

Orthonormal Family Theory

Partial Differential Equation Solution Strategies

Separation of Variables

Boundary Value Problems

Appearance of Fourier Series